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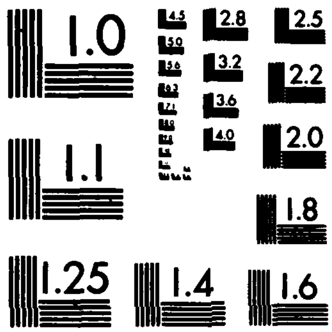

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THE BEHAVIOR OF THE ATMOSPHERE IN THE DESERT PLANETARY BOUNDARY LAYER

Louis Berkofsky
The Jacob Blaustein Institute for Desert Research
Ben-Gurion University of the Negev
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30 June 1983

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A general system of vertically integrated equations, including a dust concentration equation, has been derived. The boundary layer is divided into a constant flux layer, a transition layer, and an inversion layer. A novel feature of the model is the inclusion of equations to predict temperature and moisture lapse rates. The inversion height is also a function of time.

A simplified version of the model has been tested with respect to its ability to predict the evolution of the inversion height and of the dust concentration. The results show very reasonable evolutions. The inversion

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height is lowest in the early morning, reaching a maximum in the late afternoon. The dust concentration near the ground is highest in the early morning, when the air is most stable, and lowest in mid-afternoon, when the atmosphere is least stable. These results, obtained for various particle sizes, are highly dependent upon the form of the mesoscale vertical velocity at the base of the inversion.

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PREFACE

The research described in this report was conducted by personnel of the Jacob Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer, Israel from 1 July 1982 to 30 June 1983 under Grant No. AFOSR-82-0285 to the Ben-Gurion University of the Negev, Research and Development Authority, P.O. Box 1025, Beersheva, Israel.

Participating personnel concerned with the tasks described in this report include Prof. Louis Berkofsky, Principal Investigator, Dr. Avraham Zangvil, Research Associate, Ms. Ruth Hildebrand, Engineer-Programmer, Ms. Andrea Molod, Meteorologist-Programmer, and Ms. Perla Druian, Meteorologist-Programmer.

Observational data used in this study were obtained from the Institute's micrometeorological (4 m) tower, and from the radiation center, collected on a data logger and analyzed in the laboratory. Dust data were obtained from the Institute's Size Selective Inlet High Volume Sampler.

The Director of the Institute during the conduct of this study was Prof. Amos Richmond.

This report should be cited as follows:

Berkofsky, L., 1983. "The Behavior of the Atmosphere In the Desert Planetary Boundary Layer," Final Scientific Report, prepared by the Desert Meteorology Unit of the Jacob Blaustein Institute for Desert Research for the U.S. Air Force Office of Scientific Research, AFSC, Bolling AFB, D.C., 20332.

Introduction

In the previous Final Scientific Report (Berkofsky, 1982) we discussed the physical basis for the formulation of the Boundary Layer Model. In the present report, we discuss and present some of the results obtained from various versions of the previous model, including a first attempt to include dust modelling. We then re-derive the basic equations, using potential temperature instead of moist static energy as the principal thermodynamic variable. We also allow the lapse-rate of potential temperature to vary with time. This leads to the necessity for a prediction equation for the lapse-rate, which is derived.

Some Results From the Initial Model

Numerical Tests

In order to carry out numerical integrations, we first tested a simplified version of the model with three different numerical schemes. The equations we used were

$$\frac{\partial \hat{u}}{\partial t} + \hat{A}^2 \frac{\partial \hat{u}^2}{\partial x} - f(\hat{v} - \hat{v}_g) = \frac{[A(h+\delta)-1]}{(h-k)} \hat{u} \frac{\partial h}{\partial t} - \frac{C_d |\hat{V}| A_k^2 \hat{u}}{(h-k)} \quad (1)$$

$$\frac{\partial \hat{v}}{\partial t} + \hat{A}^2 \frac{\partial}{\partial x}(\hat{u}\hat{v}) + f(\hat{u} - \hat{u}_g) = \frac{[A(h+\delta)-1]}{(h-k)} \hat{v} \frac{\partial h}{\partial t} - \frac{C_d |\hat{V}| A_k^2 \hat{v}}{(h-k)} \quad (2)$$

$$\frac{\partial h}{\partial t} = \frac{A_1 C_{HO} A_k \hat{u} (T_{GR} - T_k)}{(T_{kI} - T_k) + (\gamma_1 - \gamma)(h-k)} \quad (3)$$

The symbols are defined in the previous report. These are very similar to the "Shallow Water" equations. We tried the usual "Leap-Frog" scheme, which, for an equation of the form

$$\frac{\partial \phi}{\partial t} + c \frac{\partial \phi}{\partial x} = 0, \tag{4}$$

c = constant

is

$$\left.\begin{aligned} \phi_j^{n+1} &= \phi_j^{n-1} + \frac{c\Delta t}{\Delta x}\left(\phi_{j+1}^{n} - \phi_{j-1}^{n}\right) \\ \phi_j^{1} &= \phi_j^{0} + \frac{c\Delta t}{2\Delta x}\left(\phi_{j+1}^{0} - \phi_{j-1}^{0}\right) \end{aligned}\right\} \tag{5}$$

The "Lax-Wendroff" scheme is

$$\left.\begin{aligned} \phi_j^{n+\frac{1}{2}} &= \frac{\left(\phi_{j+1}^{n} + \phi_{j-1}^{n}\right)}{2} + c\frac{\Delta t}{2\Delta x}\left(\phi_{j+1}^{n} - \phi_{j-1}^{n}\right) \\ \phi_j^{n+1} &= \frac{\left(\phi_{j+1}^{n} + \phi_{j-1}^{n}\right)}{2} + \frac{c\Delta t}{\Delta x}\left(\phi_{j+1}^{n+\frac{1}{2}} - \phi_{j-1}^{n+\frac{1}{2}}\right) \end{aligned}\right\} \tag{6}$$

The "Split-Explicit" scheme is much more complicated numerically, but essentially consists of three stages:

a) An adjustment stage, in which only the time-dependent geostrophic adjustment terms are integrated for several fractions of time steps.

b) An advective stage, in which only the time-dependent advective terms are integrated for one time-step, using the results of a) as initial conditions.

c) A friction stage, in which only the time-dependent friction terms are integrated for one time-step, using the results of b) as initial conditions.

Our results show that there is very little difference among the three schemes, except for a possible slight improvement with the "split-explicit." However, it may not be worth the extra computer time to continue with this method. This is still being investigated. If we abandon the "split-explicit," we will adopt the "Lax-Wendroff," as it has desirable stability characteristics.

Dust Concentration Tests

We have tested a simple version of a dust concentration model. In that version, we use a coupled inversion height and dust concentration equation, and feed in whatever other variables are needed. A complete description of the model, and the derivation of the inversion height and dust concentration equations, is given in the attached reprint (Berkofsky, 1982).

In that reprint, we derived the following equation for the variation of inversion height with time:

$$\frac{\partial h}{\partial t} = w_h + \frac{A_1 C_{H0} u_h (T_{GR} - T_h)}{[r_1 - r(h)](h - h) + (T_{hI} - T_h)} \tag{7}$$

The results are highly dependent upon W_h, the meso-scale vertical velocity, as is seen from Figs. 5 and 8 of the attached paper. There are several ways of resolving the problem of obtaining suitable values of W_h. One is to interface the boundary layer model with a free-air mesoscale model. A second method is based on the following considerations.

The continuity equation when used in Boussinesq form, i.e.,

$$\nabla \cdot \rho \mathbf{V} = 0 \tag{8}$$

may lead to spurious mass losses (Feliks and Huss, 1982). To avoid this, the continuity equation may be written

$$\nabla \cdot \rho \mathbf{V} \simeq \frac{p_0^{\frac{R}{c_p}} \, p^{\frac{c_v}{c_p}}}{R\,\theta^2} \frac{\partial \theta}{\partial t} \tag{9}$$

When this is substituted into the equation for $\frac{\partial \theta}{\partial t}$, and integrated over the depth of the transition layer (h-k), there results

$$w_h = (w_h)_B + (w_h)_H + (w_h)_S \,^{*} \tag{10}$$

where

$$\left.\begin{aligned}
(w_h)_B &= (w\theta)_k / \theta_h \\
(w_h)_H &= \left[(1+A_1)\left(\overline{w'\theta'}\right)_k - \frac{(F_h - F_k)}{c_p \bar{\rho}}\right] \Big/ \theta_h \\
(w_h)_S &= -\left[\int_k^h \theta \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz\right] \Big/ \theta_k
\end{aligned}\right\} \tag{11}$$

* A formula similar to this was derived by Lavoie (1972), but in that formula, $(W_h)_H$ is absent.

Thus, in this form, W_h depends upon the value of W at the lower boundary, $(W_h)_B$, upon the convective heating and divergence of flux of net radiation $(W_h)_H$, and upon the integral of the horizontal divergence, $(W_h)_S$. Preliminary estimates indicate that $(W_h)_S$ is the dominant term. But the above solution does reveal a diurnal variation of W_h due to $(W_h)_H$, the heating and radiation parameters. These terms may conceivably dominate in certain situations. We are continuing to test the above formulation in conjunction with Equation (7).

A third method of estimating W_h is to use a "single-station" method (Tucker, 1973). In that formulation, and using our modelling assumptions from the previous report, we find

$$W_h = \frac{-\frac{\partial T_h}{\partial t} - \gamma \frac{d}{dt}(h-h) + f/g\left[(A'B) + (AB')\right]\left[T_h + \gamma(h-h)\right]\hat{u}\hat{v} + \dot{Q}/c_p}{\gamma(z) + g/\rho c_p} \tag{12}$$

All terms except $\dot{Q}/c_p$ can be obtained from model predictions. However, W_h is needed to calculate $\frac{d}{dt}(h-h)$. This may be done interactively, i.e., assume $W_h = 0$. Solve for $\frac{d}{dt}(h)$ from the model equation. Put this into the W_h formula, get a new value, re-solve for $\frac{dh}{dt}$, etc., until convergence is reached.

An alternative to the above is simply to calculate W_h from several consectuve radiosondes, and feed this information into the model, i.e.,

$$w_h = \left[\frac{-\frac{\partial T}{\partial t} + \frac{fT}{g}\left(v\frac{\partial u}{\partial z} + u\frac{\partial v}{\partial z}\right) + \frac{\dot{Q}}{c_p}}{\frac{\partial T}{\partial z} + \frac{g}{Jc_p}}\right]_h \tag{13}$$

Re-Derivation of Model Equations

We deal with the system

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) - f(v - v_g) = -\frac{\partial}{\partial z}\overline{(u'w')} \tag{14}$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) + f(u - u_g) = -\frac{\partial}{\partial z}\overline{(v'w')} \tag{15}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{16}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial x}(u\theta) + \frac{\partial}{\partial y}(v\theta) + \frac{\partial}{\partial z}(w\theta) = H - \frac{\partial}{\partial z}\overline{(w'\theta')} \tag{17}$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}(uq) + \frac{\partial}{\partial y}(vq) + \frac{\partial}{\partial z}(wq) = -\frac{\partial}{\partial z}\overline{(w'q')} \tag{18}$$

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}\left[(w+\Omega)c\right] = -\frac{\partial}{\partial z}\overline{(w'c')} \tag{19}$$

The symbols are as previously defined:

u, v, w = components of the velocity vector.

u´, v´, w´ = turbulent components of the velocity vector.

θ = potential temperature

q = specific humidity

c = dust concentration

θ', q', c' = turbulent fluctuations of θ, q, c. $\Omega(r)$ is the velocity of c relative to the air at any one height and time. If the dust concentration at a point is distributed over particles of different sizes and fallspeeds, $\Omega(r)$ at the point is then a kind of average.

$H = \dfrac{-1}{\rho c_p} \dfrac{\partial F}{\partial z}$ = divergence of flux of net radiation

F = net radiation flux.

The bar is defined by

$$(\bar{\alpha}) = \frac{1}{\Delta x \Delta y} \iint (\alpha)\, dx\, dy \tag{20}$$

$$\alpha = \bar{\alpha} + \alpha' \tag{21}$$

where α is any scalar variable. The horizontal area $\Delta x \Delta y$ is large enough to contain the sub-grid scale phenomena, but small enough to be a fraction of the mesoscale system.

We integrate the above system (Eqs. (14)-(19) incl.) through the inversion layer, of thickness δ above the inversion layer h. Using Leibniz's rule, we find the following five Interface Conditions:

$$\left(\frac{\partial h}{\partial t} - w_h\right)\Delta u + \Delta(u^2)\frac{\partial h}{\partial x} + \Delta(uv)\frac{\partial h}{\partial y} = -(\overline{u'w'})_h \quad (2$$

$$\left(\frac{\partial h}{\partial t} - w_h\right)\Delta v + \Delta(uv)\frac{\partial h}{\partial x} + \Delta(v^2)\frac{\partial h}{\partial y} = -(\overline{v'w'})_h \quad (2$$

$$\left(\frac{\partial h}{\partial t} - w_h\right)\Delta \theta + \Delta(u\theta)\frac{\partial h}{\partial x} + \Delta(v\theta)\frac{\partial h}{\partial y} = -(\overline{w'\theta'})_h \quad (2$$

$$\left(\frac{\partial h}{\partial t} - w_h\right)\Delta q + \Delta(uq)\frac{\partial h}{\partial x} + \Delta(vq)\frac{\partial h}{\partial y} = -(\overline{q'w'})_h \quad (2$$

$$\left[\frac{\partial h}{\partial t} - (w + \Omega)_h\right]\Delta c + \Delta(uc)\frac{\partial h}{\partial x} + \Delta(vc)\frac{\partial h}{\partial y} = -(\overline{c'w'})_h \quad (2$$

In the above derivations, we have assumed that

$$\int_h^{h+\delta} \frac{1}{\rho C_p}\frac{\partial F}{\partial z}\,dz \approx \frac{1}{\rho C_p}\left(F_{h+\delta} - F_h\right) = 0$$

and

$$(\Omega + w)_{h+\delta} = (\Omega + w)_h$$

We have used the following:

Parameterizations

Winds

Surface Layer:

$$u(z) = \frac{u_*}{k_1} \ln\left(\frac{z + z_0}{z_0}\right), \quad 0 < z \leq k \tag{27}$$

u* = friction velocity, k_1 = von Karman constant

Transition Layer:

$$\left.\begin{aligned} u(x,y,z,t) &= A(z)\,\hat{u}(x,y,t) \\ v(x,y,z,t) &= B(z)\,\hat{v}(x,y,t) \end{aligned}\right\} \tag{28}$$

Here

$$\widehat{(a)} = \frac{1}{(h-k)} \int_k^h (\quad)\, dz \tag{29}$$

k = constant height of surface layer.

Temperature

$$\theta(x,y,z,t) = \theta_k(x,y,t) + r(t)(z-k) \tag{30}$$

$$\theta(x,y,z,0) = \theta_{kI}(x,y,0) + r(0)(z-k) \tag{31}$$

$$\Delta\theta = (\theta_{kI} - \theta_k) + [r(0) - r(t)](h-k) + r(t)\,\delta \tag{32}$$

Moisture

$$q(x,y,z,t) = q_h(x,y,t) + \zeta(t)(z-h) \tag{33}$$

$$q(x,y,z,0) = q_{hI}(x,y,0) + \zeta(0)(z-h) \tag{34}$$

$$\Delta q = (q_{hI} - q_h) + [\zeta(0) - \zeta(t)](h-h) + \zeta(t)\delta \tag{35}$$

Dust

$$C(x,y,z,t) = D(z)\, C_h(x,y,t) \tag{36}$$

$$\Delta C = [D(h+\delta) - D(h)]\, C_h(x,y,t) \tag{37}$$

Surface Parameterizations

$$\left.\begin{aligned} (\overline{u'w'})_h &= -C_d\, A(h)\, |V_h|\, \hat{u} \\ (\overline{v'w'})_h &= -C_d\, B(h)\, |V_h|\, \hat{v} \end{aligned}\right\} \tag{38}$$

$$(\overline{w'\theta'})_h = C_{Ho}\, A(h)\, \hat{u}\, (\theta_{GR} - \theta_h) \tag{39}$$

$$(\overline{w'q'})_h = C_{Ho}\, A(h)\, \hat{u} \left[q_{sat}(\theta_{GR}) - q_h\right] \frac{W_{GR}}{W_R} \tag{40}$$

$$
\left.\begin{aligned}
(\overline{w'c'})_h &= C_d\, A(h)\, \hat{u}\, (C_{GR} - C_h), \\
C_d &= .0005\, |V|^{\frac{1}{2}}, \quad |V| \text{ in } ms^{-1}
\end{aligned}\right\} \qquad (41)
$$

In the above, W_{GR} = ground soil moisture, θ_{GR} = ground potential temperature, and C_{GR} = ground dust concentration, must be predicted. W_k = potential saturation value of W.

Ground Albedo

$$
\left.\begin{aligned}
R_{GR} &= a + b\,\frac{W_{GR}}{W_K}, \quad b < 0 \\
&\text{a, b constants}
\end{aligned}\right\} \qquad (42)
$$

Closures

$$
(\overline{w'\theta'})_h = -A_1 (\overline{w'\theta'})_h \qquad (43)
$$

$$
(\overline{w'c'})_h = A^*(z)\,\frac{\partial c}{\partial z} \qquad (44)
$$

$$
A^*(z) = \left\{\begin{array}{ll} U_* k_1 z, & z \leq h \\ \text{constant}, & h < z \leq h \end{array}\right\} \qquad (45)
$$

We now apply the averaging operator Eq. (29) to Equations (14)-(19) inclusive, introduce the five Interface Conditions Equations (22)-(26) inclusive, all of the parameterizations and closure assumptions, Equations (28)-(30-45) inclusive, and make use of Leibniz's rule, and obtain the following system of equations.

First Equation of Motion

$$\frac{\partial \hat{u}}{\partial t} + \widehat{A^2}\frac{\partial \hat{u}^2}{\partial x} + \widehat{AB}\frac{\partial}{\partial y}(\hat{u}\hat{v}) - f(\hat{v}-\hat{v}_g) + \left[\frac{A(h+\delta)w_h - A(k)w_k}{(h-k)}\right]\hat{u} + \Big\{\left[A(k) - A(h+\delta)\right]\frac{\partial}{\partial t}\ln(h-k)$$

$$+ \left[A^2(k) + A^2(h) - \widehat{A^2} - A^2(h+\delta)\right]\hat{u}\frac{\partial}{\partial x}\ln(h-k) + \left[A(k)B(k) + A(h)B(h) - \widehat{AB} - A(h+\delta)B(h+\delta)\right]\hat{v}\frac{\partial}{\partial y}\ln(h-k)\Big\}\hat{u}$$

$$= -\frac{c_d A(k)|\hat{V}|\hat{u}}{(h-k)} \qquad (46)$$

Second Equation of Motion

$$\frac{\partial \hat{v}}{\partial t} + \widehat{AB}\frac{\partial}{\partial x}(\hat{u}\hat{v}) + \widehat{B^2}\frac{\partial}{\partial y}(\hat{v}^2) + f(\hat{u}-\hat{u}_g) + \left[\frac{B(h+\delta)w_h - B(k)w_k}{(h-k)}\right]\hat{v} + \Big\{\left[B(k) - B(h+\delta)\right]\frac{\partial}{\partial t}\ln(h-k)$$

$$+ \left[A(k)B(k) + A(h)B(h) - \widehat{AB} - A(h+\delta)B(h+\delta)\right]\hat{u}\frac{\partial}{\partial x}\ln(h-k) + \left[B^2(k) + B^2(h) - \widehat{B^2} - B^2(h+\delta)\right]\hat{v}\frac{\partial}{\partial y}\ln(h-k)\Big\}\hat{v}$$

$$= -\frac{c_d B(k)|\hat{V}|\hat{v}}{(h-k)} \qquad (47)$$

First Law of Thermodynamics

$$\frac{\partial \theta_k}{\partial t} + \frac{\partial}{\partial x}(\hat{u}\,\theta_k) + \frac{\partial}{\partial y}(\hat{v}\,\theta_k) + \frac{(w_h - w_k)}{(h-k)}\theta_k + w_h\,\Gamma(t)$$

$$+ \frac{\theta_k}{(h-k)}\left\{\frac{\partial}{\partial t}(h-k) + \left[A(k) - A(h)\right]\hat{u}\frac{\partial}{\partial x}(h-k) + \left[B(k) - B(h)\right]\hat{v}\frac{\partial}{\partial y}(h-k)\right\} = \frac{+F(k) + \hat{\rho}C_{H0}A(k)\hat{u}\,(\theta_{GR}-\theta_k)c_p}{c_p\hat{\rho}(h-k)} \qquad (48)$$

Temperature Lapse-Rate Equation

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial}{(h-k)}\left\{\frac{\partial}{\partial x}\left[\widehat{A(z-h)\hat{u}}\right] + \frac{\partial}{\partial y}\left[\widehat{B(z-h)\hat{v}}\right] - A(h)\hat{u}\frac{\partial}{\partial x}(h-k) - B(h)\hat{v}\frac{\partial}{\partial y}(h-k)\right\}\Gamma(t) = 2\frac{\left[\frac{-F(k)}{c_p\hat{\rho}} + A_1 A(k)C_{H0}\hat{u}\,(\theta_{GR}-\theta_k)\right]}{(h-k)^2}$$

Inversion Height Equation

$$\frac{\partial}{\partial t}(h-k) = w_h - \left[\frac{\left(\left[A(h+\delta)-A(h)\right]\hat{u}\left[\theta_h+r(h-k)\right]+A(h+\delta)\hat{u}\,r\delta\right)\frac{\partial h}{\partial x}+\left(\left[B(h+\delta)-B(h)\right]\hat{v}\left[\theta_h+r(h-k)\right]+B(h+\delta)\hat{v}\,r\delta\right)\frac{\partial h}{\partial y}-A(h)A_1 C_{Ho}\hat{u}\left(\theta_{GR}-\theta_h\right)}{\left(\theta_{hI}-\theta_h\right)+\left[r(0)-r(t)\right](h-k)+r(0)\delta}\right] \tag{50}$$

Continuity Equation

$$w_h = w_k - (h-k)\left(\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}\right) - \left\{\left[1-A(h)\right]\hat{u}\frac{\partial}{\partial x}(h-k) + \left[1-B(h)\right]\hat{v}\frac{\partial}{\partial y}(h-k)\right\} \tag{51}$$

Moisture Equation

$$\frac{\partial q_k}{\partial t} + \frac{\partial}{\partial x}(\hat{u}q_k) + \frac{\partial}{\partial y}(\hat{v}q_k) + \frac{(w_k-w_h)}{(h-k)}q_k + w_k\,\zeta(t) \tag{52}$$

$$+\frac{q_k}{(h-k)}\left\{\frac{\partial}{\partial t}(h-k) + \left[A(h)-A(k)\right]\hat{u}\frac{\partial}{\partial x}(h-k) + \left[B(h)-B(k)\right]\hat{v}\frac{\partial}{\partial y}(h-k)\right\} = -C_{Ho}A(k)\hat{u}\left[q_{sat}(\theta_{GR})-q_k\right]\frac{W_{GR}}{W_K}$$

Moist ure Lapse-Rate Equation

$$\frac{\partial\zeta}{\partial t} + \frac{\partial}{(h-k)}\left\{\frac{\partial}{\partial x}\left[\widehat{A(z-k)\hat{u}}\right] + \frac{\partial}{\partial y}\left[\widehat{B(z-k)\hat{v}}\right] - A(h)\hat{u}\frac{\partial}{\partial x}(h-k) - B(h)\hat{v}\frac{\partial}{\partial y}(h-k)\right\}\zeta(t) = -\frac{\left(\overline{w'q'}\right)_h}{(h-k)} \tag{53}$$

Dust Concentration Equation

$$\frac{\partial C_k}{\partial t} + \frac{\widehat{AD}}{\hat{D}}\frac{\partial}{\partial x}(C_k\hat{u}) + \frac{\widehat{BD}}{\hat{D}}\frac{\partial}{\partial y}(C_k\hat{v}) + \frac{C_k}{\hat{D}(h-k)}\left\{\left[\hat{D}-D(h)\right]\frac{\partial}{\partial t}(h-k) + \left[\widehat{AD}-A(h)D(h)\right]\hat{u}\frac{\partial}{\partial x}(h-k) + \left[\widehat{BD}-B(h)D(h)\right]\hat{v}\frac{\partial}{\partial y}(h-k) + \left[w(h)D(h)-w_k\right] + \left[D(h)-1\right]\Omega\right\}$$

$$= \frac{A^*(h)\,D'(h)\,C_k + C_d A(k)\,\hat{u}\,(C_{GR}-C_k)}{\hat{D}(h-k)} \tag{54}$$

Ground Temperature Equation

$$\frac{\partial \theta_{GR}}{\partial t} = -\frac{2\pi^{\frac{1}{2}} H_A}{\rho_s c_s d_1} - \frac{2\pi}{\tau_1}(\theta_{GR} - \theta_2) \tag{55}$$

where H_A represents the radiation balance.

Soil Moisture Equation

$$\frac{\partial W_{GR}}{\partial t} = -\frac{c_1 (E_k - P)}{\rho_w} - \frac{c_2 (W_{GR} - W_2)}{\rho_w \tau_1} \tag{56}$$

where E_k-P represents evaporation minus precipitation.

In deriving the above system, we have partitioned the radiative fluxes and the moisture fluxes in the following way: the actual First Law of Thermodynamics consisted of the sum of Equations (48) and (49). We then assumed that the change of potential temperature at the top of the surface layer is controlled mainly by the net radiation and the convective heat flux at that level, while the change of lapse rate is controlled mainly by the net radiation and the convective heat flux at inversion base level. The same type of partition led to Equations (52) and (53).

We thus have 10 prognostic equations in the 10 unknowns u, v, θ_k, γ, (h-k), q_k, ζ, C_R, θ_{GR}, W_{GR}. We have diagnostic equations for calculating $\hat{u}_g$, $\hat{v}_g$, i.e.

$$\left.\begin{aligned} \hat{u}_g &\hat{=} -\frac{R}{f}\frac{\partial}{\partial y}\left[\theta_k + r(t)\frac{(h-k)}{2}\right] \\ \hat{v}_g &\hat{=} \frac{R}{f}\frac{\partial}{\partial x}\left[\theta_k + r(t)\frac{(h-k)}{2}\right] \end{aligned}\right\} \tag{57}$$

We have parameterized $(\overline{w'\theta'})_{\ell}$, $(\overline{w'\theta'})_{k}$, $(\overline{w'\theta'})_{k}$, but not yet $(\overline{w'q'})_{k}$. We can obtain F(k) from data, but we have not specified yet the relation between F(h) and F(k), if any (see Kondratyev, 1965, p. 200). R_{GR} is parameterized. We also need W_k and C_{GR}. ρ, θ_2, W_2, W_k are prescribed. A, B, A*, D, C_d, C_{HO}, Ω are known. The lower boundary conditions have been discussed in the previous Final Report.

From the system Equations (46)-(56) inclusive, it is possible to set up a hierarchy of models, as described in the previous Final Scientific Report. Indeed, the results of the dust concentration experiment (Berkofsky, 1982) may be thought of as a zero order model.

We now describe a second version in the hierarchy.

A Meso-Scale Model

A considerable simplification of the basic system of equations results if we assume no wind shear through the inversion layer. Whether or not this is true remains to be verified. This assumption implies infinite Richardson number, making the inversion layer very stable with respect to shear:

We also assume that the atmosphere is dry and that there is no wind shear in the transition layer. There is actually some basis for the latter assumption, especially during daylight hours (Brown, 1974; Martner and Marwitz, 1982).

With these assumptions, the equations are

$$\frac{\partial \hat{u}}{\partial t} + \frac{\partial}{\partial x}(\hat{u}^2) + \frac{\partial}{\partial y}(\hat{u}\hat{v}) - f(\hat{v} - \hat{v}_g) + \frac{(w_\ell - w_k)}{(\ell - k)}\hat{u} = -\frac{C_d |V_\ell| \hat{u}}{(\ell - k)} \qquad (58)$$

$$\frac{\partial \hat{v}}{\partial t} + \frac{\partial}{\partial x}(\hat{u}\,\hat{v}) + \frac{\partial}{\partial y}(\hat{v}^2) + f(\hat{u} - \hat{u}_g) + \frac{(w_h - w_k)}{(h-k)}\hat{v} = -\frac{c_d |V| \hat{v}}{(h-k)} \tag{59}$$

$$\frac{\partial \theta_k}{\partial t} + \frac{\partial}{\partial x}(\hat{u}\,\theta_k) + \frac{\partial}{\partial y}(\hat{v}\,\theta_k) + \frac{(w_h - w_k)}{(h-k)}\theta_k + w_h\, r(t) + \frac{\theta_k}{(h-k)}\frac{\partial}{\partial t}(h-k) = \frac{+F(k) + \hat{\rho} C_{Ho}\,\hat{u}\,(\theta_{GR} - \theta_k) c_p}{c_p \hat{\rho}\,(h-k)} \tag{60}$$

$$\frac{\partial r}{\partial t} + \left\{\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right) - \frac{\left[\hat{u}\frac{\partial}{\partial x}(h-k) + \hat{v}\frac{\partial}{\partial y}(h-k)\right]}{(h-k)}\right\} r(t) = 2\left[\frac{\frac{-F(k)}{c_p \hat{\rho}} + A_1 C_{Ho}\,\hat{u}\,(\theta_{GR} - \theta_k)}{(h-k)^2}\right] \tag{61}$$

$$\frac{\partial}{\partial t}(h-k) = w_h - \left\{\frac{\left[r(t)\,\delta\left(\hat{u}\frac{\partial h}{\partial x} + \hat{v}\frac{\partial h}{\partial y}\right)\right] - A_1 C_{Ho}\,\hat{u}\,(\theta_{GR} - \theta_k)}{(\theta_{kI} - \theta_k) + [r(0) - r(t)](h-k) + r(0)\,\delta}\right\} \tag{62}$$

$$\left.\begin{aligned}
w_k &= -(h-k)\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right) - \frac{u_*}{k_1}\ln\left(\frac{z+z_0}{z_0}\right), \quad z_T \le k \\
w_k &= -\hat{u}\frac{\partial z_T}{\partial x} - \hat{v}\frac{\partial z_T}{\partial y} - (h-k)\left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\right), \quad z_T > k \\
w_k &= -\frac{u_*}{k_1}\ln\left(\frac{z_T+z_0}{z_0}\right)\frac{\partial z_T}{\partial x}, \quad z_T = k
\end{aligned}\right\} \tag{63}$$

$$\frac{\partial C_k}{\partial t} + \frac{\partial}{\partial x}(\hat{u}\,C_k) + \frac{\partial}{\partial y}(\hat{v}\,C_k) + \left\{\begin{aligned}&[\hat{D} - D(k)]\left[\frac{\partial}{\partial t}(h-k) + \hat{u}\frac{\partial}{\partial x}(h-k) + \hat{v}\frac{\partial}{\partial y}(h-k)\right] \\ &+ [w_h D(k) - w_k] + [D(k) - 1]\,\Omega\end{aligned}\right\}\frac{C_k}{\hat{D}(h-k)}$$

$$= \frac{A^*(k)\,D'(k)\,C_k + C_d\,\hat{u}\,(C_{GR} - C_k)}{\hat{D}(h-k)} \tag{64}$$

$$\frac{\partial \theta_{GR}}{\partial t} = -\frac{2\pi^{\frac{1}{2}}}{\rho_s C_s d_1} H_A - \frac{2\pi}{\tau_1}(\theta_{GR} - \theta_2) \tag{65}$$

We plan to integrate the above system - Equations (57)-(65) inclusive- over a horizontal region large enough to encompass a substantial fraction of the desert in this region, with grid spacing as yet to be determined. The lateral boundary conditions will be of the "one-way interactive" type, i.e., the data from any outer "coarse" model will be used to update conditions at inflow points of the fine model. At outflow points an open boundary condition is needed; that is, a boundary condition which allows phenomena generated in the domain of interest to pass through the boundary without undergoing significant distortion and without influencing the interior solution. Such a boundary condition is the Sommerfeld radiation condition (Orlanski, 1976; Miller and Thorpe, 1981). This is not difficult to apply and is very effective in removing boundary "noise" and reflection.

We shall use the Lax-Wendroff numerical scheme, given by Equations (6). We shall apply these on a staggered grid, with the advective form of the equations at the half time steps, and the flux form at the full time steps (Phillips, 1979).

The method of data initialization has not yet been decided, as little has been developed at this point for desert and tropical regions.

Conclusions

We have re-derived a general system of vertically integrated equations for the desert planetary boundary layer, including an equation for prediction of dust concentration. Innovative features of the revised model are the derivation of equations for prediction of lapse rate of temperature and moisture, respectively.

A test of a zero order model, in which inversion height and dust concentration are predicted, while other data are fed in, leads to what appear to be realistic results. These results were highly dependent upon the time variation of meso-scale vertical velocity, which was prescribed. Several methods for obtaining this vertical velocity from observed data are proposed.

We have derived another system in the model hierarchy. This system is two-dimensional in space, is representative of a dry atmosphere, with no wind shear across the inversion layer. This system will be integrated over a limited area of the desert, using a fine mesh with Sommerfeld radiation outflow conditions. Data initialization techniques peculiar to the desert will be used.

Recommendations

We have gathered some ground-based dust data, using a Sierra Instruments Size Selective Inlet High Volume Cascade Impactor. After much difficulty with shredding of the filter paper (substrates), we discovered that teflon backed substrates are now produced in the U.S., and we plan to continue our dust measurements using these.

We are taking radiosonde and pilot balloon observations, and plan to put into operation a tethersonde. We operate radiation devices on a regular basis, and do intermittent heat budget studies. All of the above will be used to determine various parameters and profiles for the model being tested.

We expect to interact with other people in Israel to study the effects of meteorology on transmission of radiation and imaging through natural dusty environments.

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A Heuristic Investigation to Evaluate the Feasibility of Developing a Desert Dust Prediction Model[1]

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ABSTRACT

A boundary layer model, to investigate various aspects of desert circulation, has been devised. The planetary boundary layer is divided into a surface (constant flux) layer, a transition layer and an inversion layer. The basic equations (motion, continuity, first law of thermodynamics, dust concentration equation) are integrated with respect to height from the bottom of the transition layer up to the top of the boundary layer. It is assumed that this height coincides with that of the inversion, which is allowed to vary in space and time. Interface conditions are derived by integration of the equations across the inversion layer. These may be used to eliminate the momentum, moisture and dust concentration turbulent fluxes from the vertically integrated equations. The turbulent heat flux is eliminated from the first law by a closure assumption, relating the turbulent flux at the base of the inversion to that at the bottom of the transition layer. Modelling assumptions, specifying the variation with height of wind, temperature and dust concentration, are used in the vertically integrated equations. The resulting equations predict *the horizontal wind components at a* "mean" level, and the temperature and dust concentration at the bottom of the transition layer.

A simplified version of the model, in which advective terms are neglected, couples the dust concentration and inversion height equations. In the former, the time variation of the dust concentration depends upon the difference between sedimentation and turbulent diffusion of dust, and upon the time variation of the inversion height. The latter depends upon the turbulent transport of heat from the ground, upon the difference between the initial temperature and temperature at a later time, and upon the vertical velocity of the larger scale motions at interface height. A numerical integration of this system describes the evolution of the inversion height, its effect on the dust concentration evolution, as well as the evolution of the dust concentration itself.

1. Introduction

The problem of dust transport has been receiving increasing attention in recent years. Of particular interest has been the transport of Saharan dust by large-scale atmospheric circulations (Morales, 1979).

Smaller scale transport is also of interest for a variety of reasons. Among these are the siting of buildings, location of agricultural areas, effects on health, effects on aviation, effects on machinery, and effects on light transmission by various atmospheric probes.

To model several scales simultaneously is a formidable task. The few studies that have been made confine themselves to a single scale. Since we are interested in the dust problem as it concerns future settlement of the Negev (Berkofsky, *et al.*, 1981), we shall confine ourselves to the mesoscale, commensurate with the size of the Negev.

The Negev is under the influence of the descending branch of a subtropical high pressure cell. As a result, subsidence inversions exist on the average 222 days per year (Shaia and Jaffe, 1976). Even when no dust storm influences the region, the loess which is picked up by the wind is trapped within the boundary layer. To be able to predict the daily variation of dust concentration requires a method to predict the variation of inversion height.

2. The model

Although we have developed a model which is completely time and space dependent, we shall deal here with a very simple version, mainly as a testing ground for the complete model.

The method consists of averaging vertically the basic equations, and introducing modelling assumptions in the vertical, thereby eliminating the vertical dimension. At the same time, various parameterizations are introduced for the fluxes of dust and heat.

We divide the atmospheric boundary layer into layers, as shown in Fig. 1. Within the surface layer of height $z = k$, we assume that the wind obeys the constant flux formula. The height of the transition layer is $h - k$. The thickness of the inversion layer is δ.

The temperature variations above the surface layer

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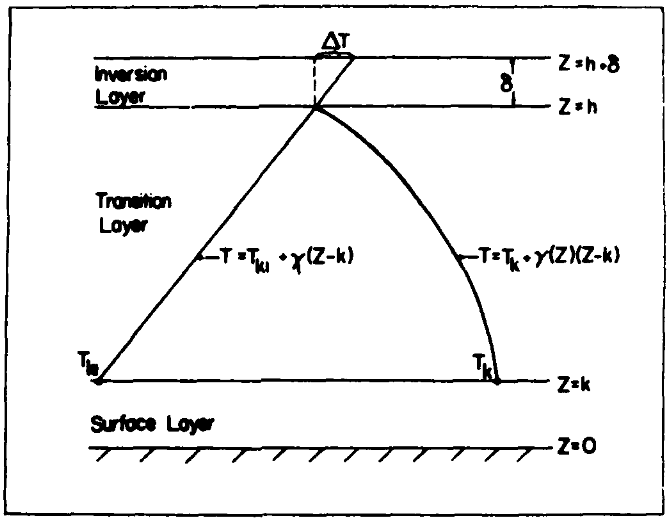

FIG. 1. Schematic of planetary boundary layer.

are defined by

$$T(z, t) = T_k(t) + \gamma(z)(z - k), \qquad (1)$$

$$T(z, 0) = T_{kI} + \gamma_1(z - k), \qquad (2)$$

where γ_1, a constant, is the increase of temperature with height at $t = 0$ and also above base of inversion at all times, $\gamma(z)$ is the falloff of temperature with height in transition layer, $T_k(t)$ the temperature at $z = k$, and T_{kI} the temperature at height $z = k$ at initial time.

We then find that the inversion strength is given by

$$\Delta T = (T_{kI} - T_k) + [\gamma_1 - \gamma(h)](h - k) + \gamma_1\delta. \quad (3)$$

In terms of potential temperature θ, we have

$$\Delta\theta = (\theta_{kI} - \theta_k) + \left(\frac{p_0}{p_h}\right)^\kappa [\gamma_1 - \gamma(h)](h - k) + \left(\frac{p_0}{p_h}\right)^\kappa \gamma_1\delta, \quad (4)$$

where p is pressure, $\kappa = R/c_p$, R is the gas constant, and c_p the specific heat at constant pressure.

The variation of dust concentration with height is given by

$$C(z, t) = D(z)C_k(t), \qquad (5)$$

$$D(z) = (z/k)^{\Omega/(k_1 U_*)}. \qquad (6)$$

Here k_1 is the von Kármán constant (0.4), U_* the friction velocity, and $\Omega(r)$ the sedimentation or settling velocity of dust particles of radius r.

Gillette and Goodwin (1974) solved a one-dimensional steady-state equation in which vertical turbulent diffusion of dust balanced sedimentation in the surface layer. Their solution was of the form shown as Eq. (5), with the time-dependence omitted, and in which $D(z)$ was given by (6), except that their result used Z_0, the roughness height, instead of k. We have assumed that a time-dependent relation of this form exists throughout the transition layer. The form of $D(z)$ should be determined from data, which are at present unavailable. For the time being we use the form above, as a modelling assumption.

The equation for the rate of change of dust concentration in a vertical column is

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial z}[C(w + \Omega)] = -\frac{\partial}{\partial z}(\overline{w'C'}), \qquad (7)$$

where w is the vertical velocity of the air. The velocity of C (the dust content of the air) relative to the air at any one height and time is represented by $\Omega(r)$, a negative quantity since it is downward. If the dust concentration at a point is distributed over particles of different sizes and fallspeeds, $\Omega(r)$ at that point is then a kind of average. w' and C' are deviations from their means, and the bar is a horizontal average over a sufficiently large area.

Eq. (7), it is noted, retains the vertical motion term, but omits the horizontal advection. Thus, the omission of $\nabla \cdot (C\mathbf{V}) = C\nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla C$ and $\nabla \cdot (\theta\mathbf{V}) = \theta\nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla\theta$ in the first law of thermodynamics, to be considered later (here $\mathbf{V}$ is the horizontal wind vector), is tantamount to assuming that advection and horizontal divergence are in balance. The horizontal divergence need not be zero, so that the vertical motion term need not be neglected. Our aim, at this stage, is not to develop an operational model, but to test the feasibility of doing so. In steady-state models, like those of Gillette and Goodwin (1974) the vertical motion term is correctly omitted, since they assumed zero horizontal divergence. Here we are interested in seeing the effect of vertical motion on the evolution of inversion height and dust concentration, without

having to solve the complete set of primitive equations. For this reason, we include the vertical motion term in the dust concentration and in the first law of thermodynamics. For purposes of computation in the final equations, we shall assume a distribution with time of vertical velocity at inversion height.

We apply the operator

$$(\hat{\ }) = \frac{1}{h-k}\int_k^h (\ )dz \tag{8}$$

to Eq. (7), and introduce Eq. (5). Integrating by parts using Leibniz's rule, and recalling that $h = h(t)$, yields

$$\frac{\partial C_k}{\partial t} = \frac{C_k[D(h) - \hat{D}]\dfrac{\partial h}{\partial t} + [(w_k + \Omega) - D(h)(w_h + \Omega)]C_k - (\overline{w'C'})_h + (\overline{w'C'})_k}{\hat{D}(h-k)}. \tag{9}$$

We parameterize $(\overline{w'C'})_h$ by means of

$$-(\overline{w'C'}) = A^*(z)\frac{\partial C}{\partial z}, \tag{10}$$

i.e.,

$$-(\overline{w'C'})_h = A^*(h)\left(\frac{\partial D}{\partial z}\right)_h C_k, \tag{11}$$

where $A^*(z)$ is the coefficient of eddy diffusion, given by

$$\left.\begin{array}{l} A^*(z) = U_* k_1 z, \quad z \leq k \\ \text{[see Gillette and Goodwin (1974)]} \\ A^*(z) = \text{constant}, \quad k < z \leq h \end{array}\right\} \tag{12}$$

and $(\overline{w'C'})_k$ by means of

$$(\overline{w'C'})_k = C_d u_k (C_{GR} - C_k). \tag{13}$$

Here C_{GR} is the concentration in a thin layer near the ground and u_k is the east–west component of wind at $z = k$. This parameterization for dust concentration, analogous to that for momentum and heat, provides the mechanism for vertical turbulent transport of dust. In (13), C_d is the transfer coefficient for dust. For sufficiently small particles it may be assumed that C_d is the same as the transfer coefficient for momentum (Chepil and Woodruff, 1957). Then

$$C_d = 0.0005\ V^{1/2}, \quad V \text{ in m s}^{-1}. \tag{14}$$

Then, finally

$$\frac{\partial C_k}{\partial t} = \frac{[D(h) - \hat{D}]C_k\dfrac{\partial h}{\partial t} + \{\overbrace{[(w_k + \Omega) - D(h)(w_h + \Omega)]}^{\text{sedimentation}} + \overbrace{A^*D'(h)\}C_k + C_d u_k(C_{GR} - C_k)}^{\text{diffusion}}}{\hat{D}(h-k)}. \tag{15}$$

The terms responsible for sedimentation and diffusion are labelled. Notice that the time rate of change of inversion height has an effect on the concentration tendency. But this effect is small. However, the quantity $h - k$ in the denominator strongly influences $\partial C_k/\partial t$ since $h - k$ goes through a large variation during the course of a day.

The sedimentation velocity $\Omega(r)$ depends upon the radius of the dust particle. Since the above equation refers to a single value of Ω, it predicts the concentration as if there were a single particle size within the distribution. To account for other sizes would require solving the equation for other values of $\Omega(r)$, and summing the results. Of course, initial values of C_k would have to be known for various size particles. If we are concerned with the case of atmospheric dust particles [see Fig. 2 (after Bagnold, 1954)], we see that these are sufficiently small so that they have only a slight sedimentation velocity. Thus, what is likely is that the heavier particles fall out, while the smaller particles, which have smaller sedimentation velocities, require only a slight upward motion or upward transfer to keep them in suspension.

The derivation of an appropriate inversion height equation has been the subject of intensive investigation for many years (see, for example, Tennekes and Driedonks, 1981; Smeda, 1979; Stull, 1973). Tennekes (1973), Betts (1973) and Carson (1973) have assumed the entrainment rate of heat from the inversion layer is such that

$$-(\overline{w'\theta'})_h = A_1(\overline{w'\theta'})_s, \tag{16}$$

where $(\overline{w'\theta'})_s$ in the surface value. We shall use this relation in the form

$$-(\overline{w'\theta'})_h = A_1(\overline{w'\theta'})_k. \tag{17}$$

We make use of this relation in the "interface condition" derived in the following way. The first law of thermodynamics can be written

$$\frac{\partial\theta}{\partial t} + \frac{\partial}{\partial z}(w\theta) = Q - \frac{\partial}{\partial z}(\overline{w'\theta'}), \tag{18}$$

where Q is the divergence of the flux of net radiation. We integrate this equation between the levels h and

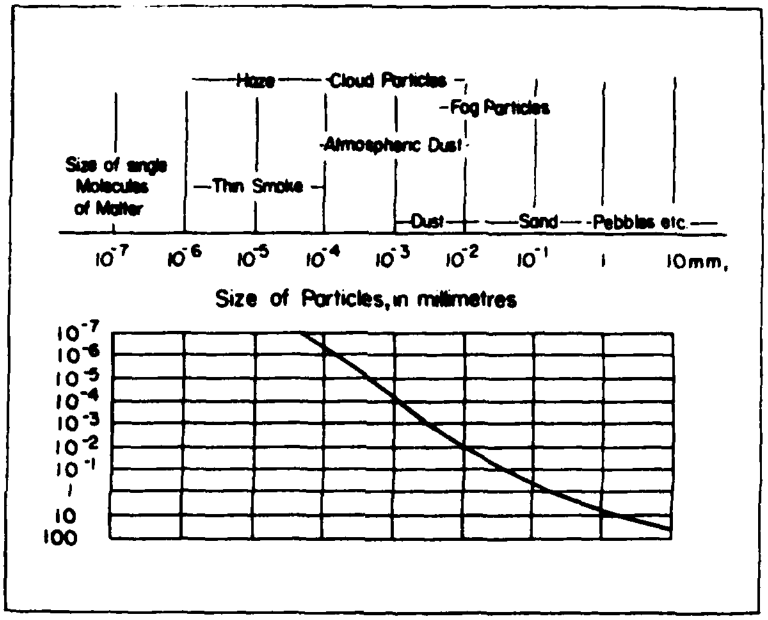

FIG. 2. Relative size and fall rate (m s^{-1}, ordinate of lower figure) of small particles (after Bagnold, 1954).

$h + \delta$, and let $\delta \to 0$. Then

$$\left(\frac{\partial h}{\partial t} - w_h\right)\Delta\theta = -(\overline{w'\theta'})_h\,. \tag{19}$$

Using

$$(\overline{w'\theta'})_k = C_{HO}u_k(\theta_{GR} - \theta_k), \tag{20}$$

where θ_{GR} is the potential temperature at the ground, and C_{HO} the heat transfer coefficient, together with Eqs. (19) and (4), we obtain

$$\frac{\partial h}{\partial t} = w_h + \frac{A_1 C_{HO} u_k(\theta_{GR} - \theta_k)}{\left(\frac{p_0}{p_h}\right)^{\kappa}[\gamma_1 - \gamma(h)](h - k) + (\theta_{kI} - \theta_k) + \left(\frac{p_0}{p_h}\right)^{\kappa}\gamma_1\delta}, \tag{21}$$

which is similar to the Tennekes (1973) model.

Eqs. (21) and (15) are two of the equations of the complete prediction scheme. To complete the system, we would need five more equations: the first equation of motion for prediction of u_k, the first law of thermodynamics for prediction of T_k, a ground temperature equation for prediction of T_{GR}, the equation of continuity for calculation of w_h, and an equation for prediction of dust concentration at the ground C_{GR}.

We have, in fact, derived this system, using the same approach as above. Here we wish to discover whether it is possible to obtain meaningful results from an abbreviated version, before going ahead with the full model. Thus, wherever possible, we shall feed in observed data, such as u_k, T_{GR}, T_k. The problem of obtaining observed data for C_{GR} is more complicated, and more properly belongs to the realm of sedimentology. However, *in discussions with Prof.* P. R. Owen (1981, private communication), the assumption of setting C_{GR} = constant for 24 h is reasonable. There is obviously much room for improvement of this assumption, and we do not make any strong claims for it at this point.

TABLE 1. Data taken at Sede Boqer, Israel, 18 June 1979. The W_h values are assumed.

Time (LST)	$(T_{kI} - T_k)$ (°C)	$u_k(T_{GR} - T_k)$ (cm s^{-1} °C)	u_k (cm s^{-1})	w_h (cm s^{-1})
0900	0.00	850	291	1.0
1000	−1.95	4738	480	1.0
1100	−3.35	5440	515	1.5
1200	−4.35	5566	550	1.75
1300	−5.75	5470	560	2.0
1400	−5.05	5220	570	2.0
1500	−4.75	3540	695	2.0
1600	−4.05	2360	820	1.0
1700	−2.50	1320	680	0.0
1800	−.89	760	540	−1.0
1900	.95	340	418	−1.0
2000	2.65	20	295	−1.0
2100	4.00	−220	223	−1.5
2200	5.27	−330	151	−2.0
2300	6.35	−80	204	−2.0
0000	7.25	18	258	−2.0
0100	8.05	−200	223.5	−2.0
0200	8.88	−580	189	−2.0
0300	9.95	−590	190	−1.0
0400	10.55	−535	192	−1.0
0500	10.45	−400	166	−1.0
0600	9.58	−220	140	0.0
0700	6.55	80	120	0.25
0800	2.70	570	101	0.50

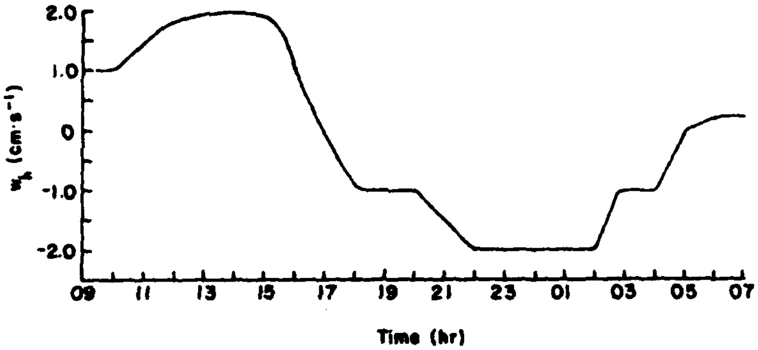

FIG. 3. Assumed distribution of w_h.

Since $(p_0/p_h)^\kappa \approx 1$ for a range of values of h, we use that here. Further, since $(p_0/p_h)^\kappa \approx 1$ and $\gamma_1\delta \ll (\theta_{kl} - \theta_k)$ for reasonable values of both these quantities, we may write (21) as

$$\frac{\partial h}{\partial t} = w_h + \frac{A_1 C_{HO} u_k (T_{GR} - T_k)}{[\gamma_1 - \gamma(h)](h - k) + (T_{kl} - T_k)}, \quad (22)$$

where T_{GR} is ground temperature.

Eqs. (22) and (15) comprise the test scheme, given certain input variables and constants.

3. Calculations and results

Initial values of $h = 300$ m at 0900 LST and $C_k = 100$ μg m^{-3} are assumed. The latter value is taken from Junge (1979) as representative of Saharan air. We use $A_1 = 0.2$, $\gamma_1 = 0.5 \times 10^{-4}$ K cm^{-1}, $\gamma(h) = -0.5 \times 10^{-4}$ K cm^{-1}, $C_{HO} = 1.695 \times 10^{-3}$, $A^*(h) = 1.6 \times 10^4$ cm^2 s^{-1}, $C_{GR} = 100$ μg m^{-3} = constant, $k = 20$ m. We also assume $w_k = 0.1 w_h$. Table 1 gives a tabulation of $T_{kl} - T_k$; $u_k(T_{GR} - T_k)$; u_k; and an assumed distribution w_h (Fig. 3) for Sede Boqer, Israel for 18 June 1979. The value of $A^*(h)$ is obtained from Eq. (12) with $U_* = 20$ cm s^{-1}, $k_1 = 0.4$, $z = 20$ m (top of surface layer).

The distribution of w_h is problematical. These values should come (as stated earlier) from solutions of the continuity equation. For sensitivity test purposes, we have assumed rising motion during daylight hours, and sinking motion from late afternoon until sunrise. This is certainly qualitatively correct on a clear summer day, but the values could be changed somewhat without greatly affecting the shapes of the calculated curves for h and C_k. The notable difference occurs when $w_h = 0$ at all times.

We have carried out calculations for three different values of $\Omega(r)$: -1.0, -0.1 and -0.01 cm s^{-1}. As can be seen from Fig. 2, these represent particles of radii 10^{-2}, 10^{-3} and 4×10^{-4} mm, i.e., approximately 10, 1 and 0.4 μm—all within the range of atmospheric dust particles. Fig. 4 (after Gillette) shows a graph of Ω/U_* vs. the ratio of upward fluctuations to downward fluctuations. U_* is a measure of the standard deviation of w'. We see, for the values used here ($U_* = 20$ cm s^{-1}, $\Omega = -1.0, -0.1, -0.01$ cm s^{-1}), that the ratio of upward to downward fluctuations is very close to 1. This means that particles with these radii will probably stay in suspension unless acted on by sufficiently large vertical velocity of the larger scale motions.

Fig. 5 shows the results of solving Eq. (21) with the data given. On the same figure is the march of $(\rho_k c_p)^{-1} \times$ Heat Flux (this curve is labelled HF), near the ground. Although no verification data are as yet available, it can be seen that the variation of inversion height throughout the day is reasonable. The maximum of 1051 m is reached at 1800 LST. This maximum lags that of the heat flux by ~6 h, which is in agreement with the O'Neill, Nebraska results shown in Fig. 1 of Carson (1973). Our results are very sensitive to vertical velocity variations (see Figs. 6–9).

Fig. 6 shows the results of solving Eq. (22) together with (15), for $\Omega(r) = -1.0$ cm s^{-1} and $\Omega(r) = -0.1$ cm s^{-1}. The concentration C_k decreases as the day

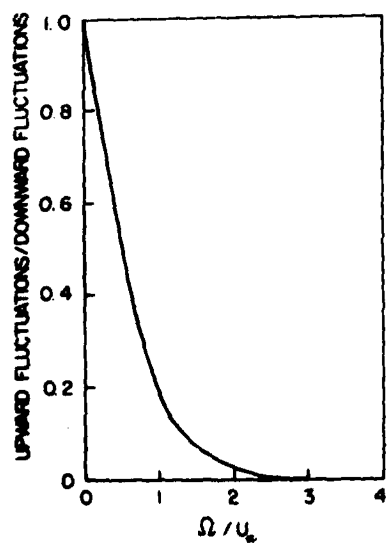

FIG. 4. Sedimentation velocity compared to vertical velocity fluctuations (after Gillette, 1979).

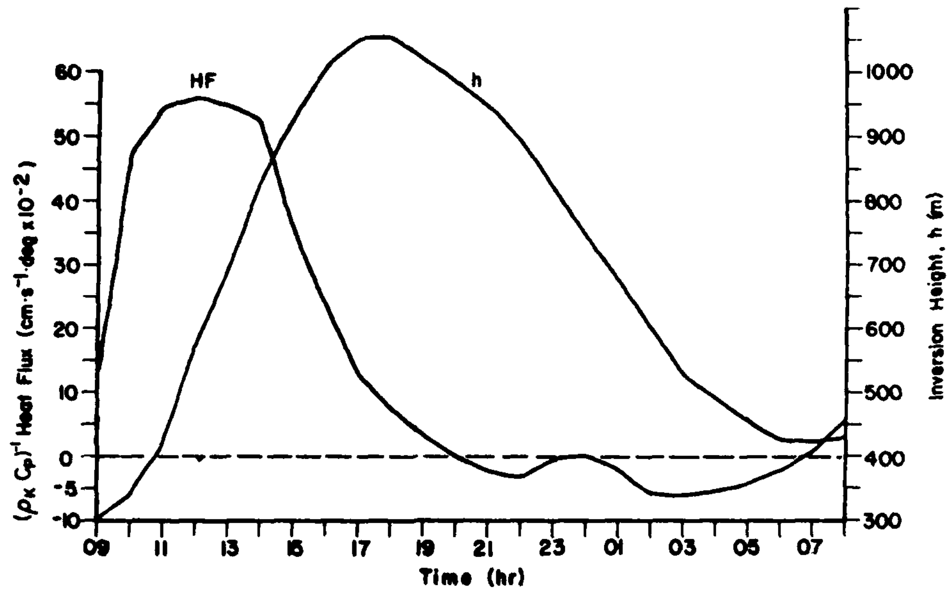

FIG. 5. Time variation of inversion height (h) and surface heat flux (HF).

progresses, reaches a minimum around 2 h before the maximum of inversion height, and then goes back to a slightly higher value than the initial one by the end of the 24 h period. What appears to be happening is that when $(w_h + \Omega) > 0$ (from 0900–1600), the particles rise. C_k decreases during that time because the inversion height increases. Even though there is an upward transfer of dust ($C_{GR} > C_k$), the increase of inversion height plus the net upward motion distributes the dust upward. After 1600, when $(w_h + \Omega) < 0$ the particles sink, and C_k increases. At the same time, there is a weak upward flux of C, i.e., $C_{GR} > C_k$, to increase C_k. Also h is decreasing, so that the dust is being "squeezed" into a smaller volume. Notice that, for $\Omega = -0.1$ cm s^{-1}, C_k stays higher than for $\Omega = -1.0$, simply because of the smaller settling velocity. The results for $\Omega = -0.01$ are almost the same as for $\Omega = -0.1$.

Fig. 7 shows the total concentration $\int_k^h C dz$ for the three values of Ω. In all cases, the total concentration increases and then decreases. This is undoubtedly due to the fact that we are forcing C_{GR} to remain constant.

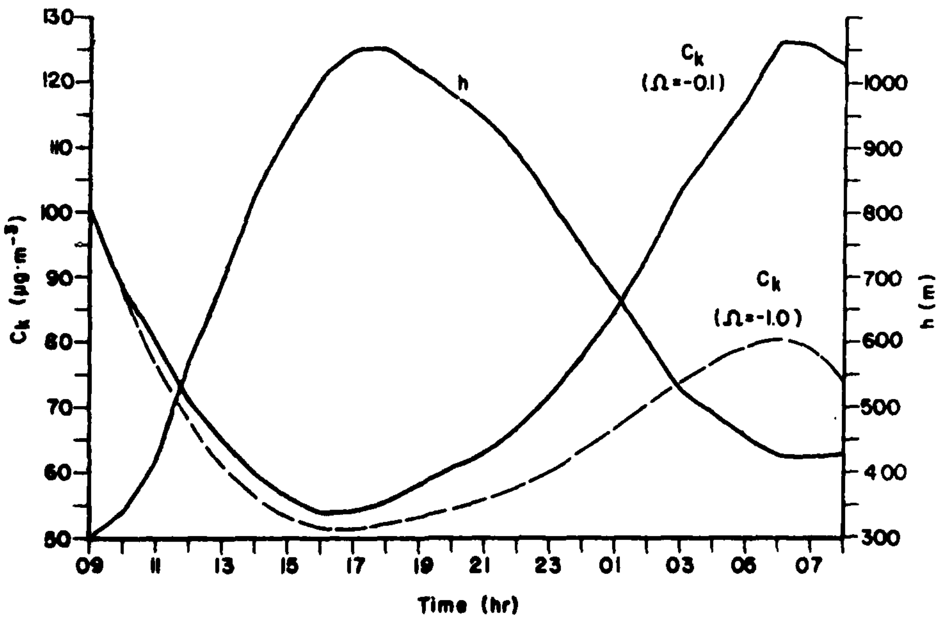

FIG. 6. Time variation of inversion height (h) and dust concentration (C_k).

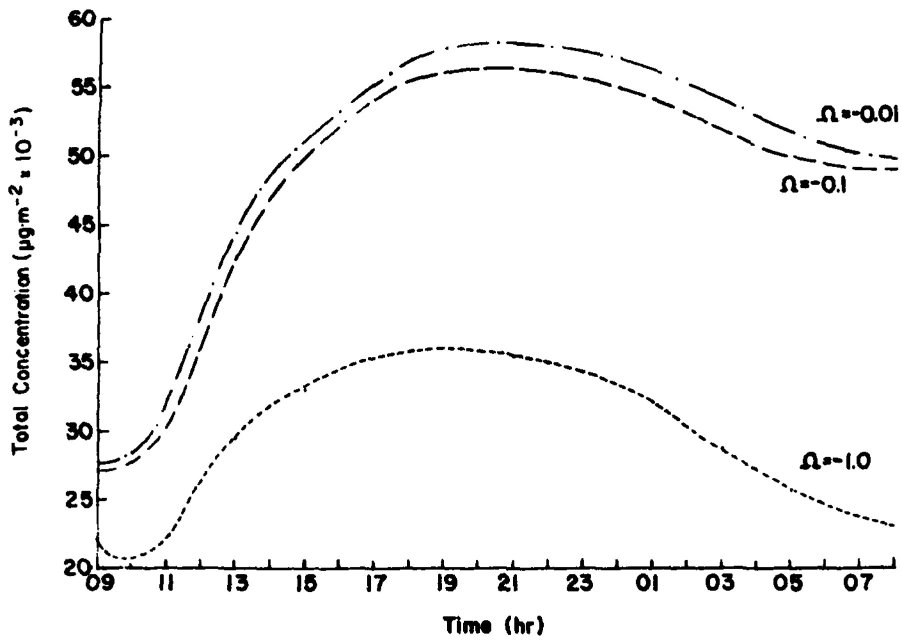

FIG. 7. Time variation of total concentration.

In actual fact, as long as there is an upward transfer ($C_{GR} > C_k$), C_{GR} should decrease. The fact that we are keeping C_{GR} constant is tantamount to assuming that a source exists to maintain the value of C_{GR}. Thus dust is being added, and the total concentration increases. Later in the day, the subsidence overcomes the upward turbulent transport, and decreases the total concentration. To account for the changes in C_{GR} requires a more accurate prediction equation than simply assuming C_{GR} to be a constant.

The lighter particles are distributed upward more easily, increase the total concentration, and stay in suspension longer. This is so because the subsidence velocity plus the sedimentation velocity of these particles is less than for the heavier particles.

Fig. 8 shows the results when $w_h = 0$ at all times. The inversion height h reaches a somewhat smaller maximum than when $w_h \neq 0$, at about the same time, and remains constant after that. The concentration C_k decreases steadily with time when $\Omega = -1.0$ cm

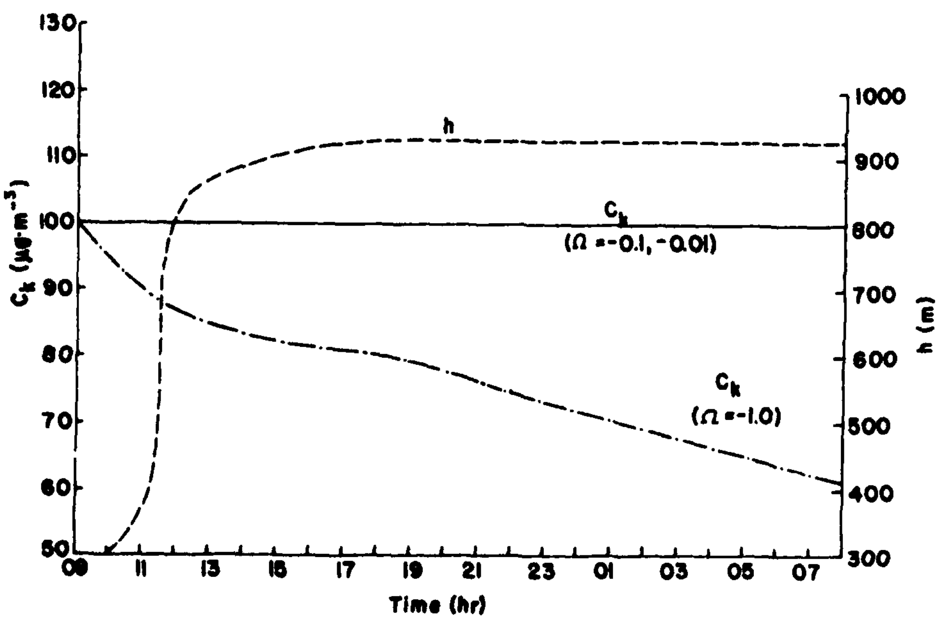

FIG. 8. Time variation of inversion height (h) and dust concentration (C_k)(w_h = 0).

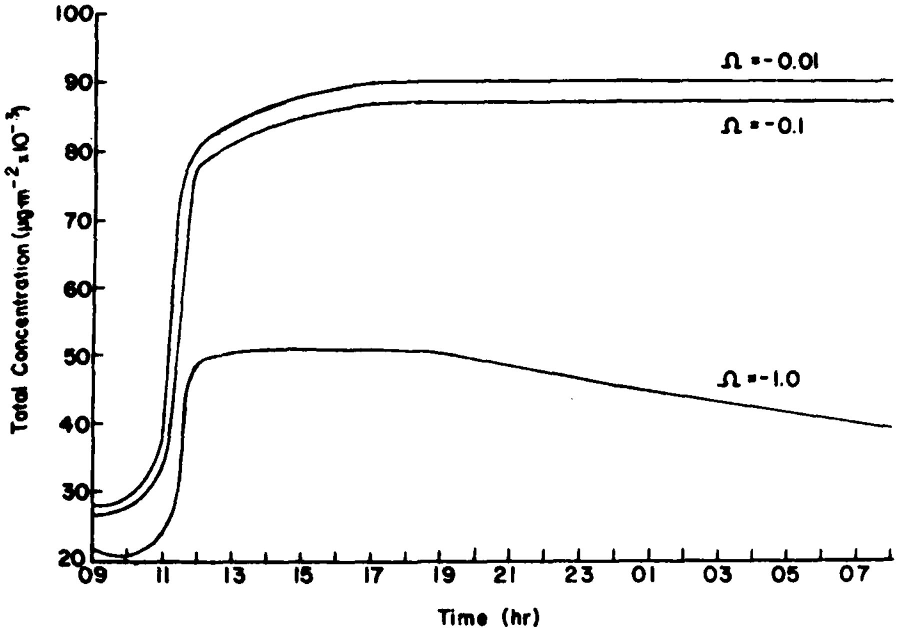

FIG. 9. Time variation of total concentration (w_h = 0).

s^{-1}, but remains constant when $\Omega = -0.1$–0.01 cm s^{-1}. This simply means that the lighter particles are essentially in suspension when not acted on by significant vertical motion. The role of subsidence in determining inversion height and concentration is clearly shown by these results.

Fig. 9 shows the total concentration for the above three values of Ω when $w_h = 0$. Similar arguments to the above can be made to explain the results.

To date, no data exist at Sede Boqer to verify any of the above results.

4. Conclusions

An extremely simplified version of a model for prediction of dust concentration has yielded what appear to be reasonable results. We are thus encouraged to continue with the full model, in which the entire interactive system (primitive equations) will be used. Consideration will also be given to possible heating effects of lower troposphere dust concentrations.

At the same time, we are developing a program for dust and inversion height measurements at our location in the Negev. We will then be able to verify present and future results, and use the data for other dust studies, as mentioned in the Introduction.

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